Supporting Information

Interval Multi-Random Factorial Programming for Coupled Farmland and Water Resources Management -- A Case Study of Songhua River Watershed, China

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IMRFP Model:

Based on the analysis mentioned above, the objective function of the IMRFP model is formulated as follows:

$$Max f^{\pm} \cong \sum_{i=1}^{4}\sum_{j=1}^{3}\sum_{t=1}^{3} BP_{ijt}^{\pm} \cdot YP_{ijt}^{\pm} \cdot P_{ijt}^{\pm} + \sum_{k=1}^{2}\sum_{j=1}^{3}\sum_{t=1}^{3} BI_{kjt}^{\pm} \cdot AI_{kjt}^{\pm} + \sum_{j=1}^{3}\sum_{t=1}^{3} BT_{jt}^{\pm} \cdot AT_{jt}^{\pm} + \sum_{j=1}^{3}\sum_{t=1}^{3} BR_{jt}^{\pm} \cdot AR_{jt}^{\pm}$$
$$- \sum_{j=1}^{3}\sum_{t=1}^{3} (CS_{jt}^{\pm} \cdot SW_{jt}^{\pm} + CG_{jt}^{\pm} \cdot GW_{jt}^{\pm}) - \sum_{k=1}^{2}\sum_{t=1}^{3} (CWI_{kt}^{\pm} \cdot \sum_{j=1}^{3} (RI_{kjt}^{\pm} \cdot AI_{kjt}^{\pm})) - \sum_{t=1}^{3} (CWT_{t}^{\pm} \cdot \sum_{j=1}^{3} (RT_{jt}^{\pm} \cdot AT_{jt}^{\pm})) - \sum_{t=1}^{3} (CWR_{t}^{\pm} \cdot \sum_{j=1}^{3} (RR_{jt}^{\pm} \cdot AR_{jt}^{\pm})) \quad \text{(S1)}$$

Constraints of the IMRFP model consist of the following inequalities.

1) Songhua River Watershed farmland availability

a) Maximum cultivation areas

$$\sum_{i=1}^{3} P_{ijt}^{\pm} \le MAXA_{jt} \quad \forall j,t \quad \text{(S2)}$$

b) Minimum cultivation areas

$$\sum_{i=1}^{3} P_{ijt}^{\pm} \ge MINA_{jt} \quad \forall j,t \quad \text{(S3)}$$

2) Songhua River Watershed water resources availability

a) Surface water availability

$$\sum_{j=1}^{3} SW_{jt}^{\pm} \le MAXS_{t}^{\pm} \quad \forall t \quad \text{(S4)}$$

b) Groundwater availability

$$\sum_{j=1}^{3} GW_{jt}^{\pm} \le MAXG_{t}^{\pm} \quad \forall t \quad \text{(S5)}$$

c) Total water resources availability

$$\sum_{i=1}^{4}\sum_{j=1}^{3} RDP_{ijt}^{\pm} \cdot P_{ijt}^{\pm} + \sum_{k=1}^{2}\sum_{j=1}^{3} AI_{kjt}^{\pm} + \sum_{j=1}^{3} AT_{jt}^{\pm} + \sum_{j=1}^{3} AR_{jt}^{\pm} \le \sum_{j=1}^{3} (SW_{jt}^{\pm} + GW_{jt}^{\pm}) \quad \forall t \tag{S6}$$

3) Songhua River Watershed water supply constraints

a) Water supply for agriculture

$$\sum_{i=1}^{4}\sum_{j=1}^{3} RDP_{ijt}^{\pm} \cdot P_{ijt}^{\pm} \le MAXWA_{t}^{\pm} \ \forall t \tag{S7}$$

b) Water supply for industry

$$\sum_{j=1}^{3} AI_{jt}^{\pm} \le MAXWI_{t}^{\pm} \quad \forall t \tag{S8}$$

c) Water supply for tourism

$$\sum_{j=1}^{3} AT_{jt}^{\pm} \le MAXWT_{t}^{\pm} \quad \forall t \tag{S9}$$

d) Water supply for household

$$\sum_{j=1}^{3} AH_{kjt}^{\pm} \le MAXWH_{t}^{\pm} \quad \forall t \tag{S10}$$

4) Songhua River Watershed wastewater treatment capacity constraints

$$\sum_{k=1}^{2}\sum_{j=1}^{3} RI_{kjt}^{\pm} \cdot AI_{jt}^{\pm} + \sum_{j=1}^{3} RT_{jt}^{\pm} \cdot AT_{jt}^{\pm} + \sum_{j=1}^{3} RP_{jt}^{\pm} \cdot AR_{jt}^{\pm} \le MAXUT_{t}^{\pm} \quad \forall t \tag{S11}$$

5) Songhua River Watershed eco-environment constraints

a) Soil erosion control

$$\sum_{i=1}^{4}\sum_{j=1}^{3} COSL_{ijt}^{\pm} \cdot P_{ijt}^{\pm} \leq MAXCSL_{t}^{\pm} \quad \forall t \tag{S12}$$

b) Nitrogen discharge control

$$\sum_{i=1}^{4}\sum_{j=1}^{3} QN_{ijt}^{\pm} \cdot P_{ijt}^{\pm} + (\sum_{k=1}^{2}\sum_{j=1}^{3} QNI_{kjt}^{\pm} \cdot AI_{kjt}^{\pm} + \sum_{j=1}^{3} QNT_{jt}^{\pm} \cdot AT_{jt}^{\pm} + \sum_{j=1}^{3} QNR_{jt}^{\pm} \cdot AR_{jt}^{\pm})(1 - NRE_{t}^{\pm}) \leq MAXTN_{t}^{\pm} \quad \forall t \tag{S13}$$

c) Phosphor discharge control

$$\sum_{i=1}^{4}\sum_{j=1}^{3} QS_{ijt}^{\pm} \cdot P_{ijt}^{\pm} + (\sum_{k=1}^{2}\sum_{j=1}^{3} QPI_{kjt}^{\pm} \cdot AI_{kjt}^{\pm} + \sum_{j=1}^{3} QPT_{jt}^{\pm} \cdot AT_{jt}^{\pm} + \sum_{j=1}^{3} QPR_{jt}^{\pm} \cdot AR_{jt}^{\pm})(1 - PRE_{t}^{\pm}) \leq MAXTP_{t}^{\pm} \quad \forall t \tag{S14}$$

where f = the expected net system benefit (\$); t = time period, t = 1, 2, 3 (where t = 1 for 2020 to 2024, 2 for 2025 to 2029, 3 for 2030 to 2034); i = the type of crops, i = 1, 2, 3,4 (where i = 1 for corn, 2 for soybean, 3 for potato, 4 for rice); j = sub-region, j = 1, 2, 3 (where j = 1 for Inner Mongolia, 2 for Jilin, 3 for Heilongjiang); k = the type of industry, k = 1, 2 (where k = 1 for metallurgical industry, 2 for food industry); $BP_{ijt}^{\pm}$ = market price of crop i in sub-region j in period t (\$/kg); $BC_{ijt}^{\pm}$ = yield of crop i in sub-region j in period t (kg/km^2); $BI_{ijt}^{\pm}$ = unit benefit of water allocated to industry k in sub-region j in period t (\$/m^3); $BT_{ijt}^{\pm}$ = unit benefit of water allocated to tourism in sub-region j in period t (\$/m^3); $BR_{ijt}^{\pm}$ = unit benefit of water allocated to household in sub-region j in period t (\$/m^3); $CS_{jt}^{\pm}$ = cost for pumping and delivering the surface water in sub-region j in period t (\$/m^3); $CG_{jt}^{\pm}$ =cost for pumping and delivering the ground water in sub-region j in period t (\$/m^3); $CWI_{kt}^{\pm}$ = treatment cost of wastewater from industry k in period t (\$/tonne); $CWT_{t}^{\pm}$ = treatment cost of wastewater from tourism industry in period t (\$/tonne); $CWR_{t}^{\pm}$ = treatment cost of wastewater from household in period t; $RI_{kjt}^{\pm}$ = unit wastewater discharge by industry k in sub-region j in period t; $RT_{kjt}^{\pm}$ = unit wastewater discharge by tourism industry in sub-region j in period t (tonne/m^3); $RR_{kjt}^{\pm}$ = unit wastewater discharge by household in sub-region j in period t (tonne/m^3); $MAXA_{jt}$ = the maximum area allocated to crop i in sub-region j in period t (km^2); $MINA_{jt}$ = the minimum area allocated to crop i in sub-region j in period t (km^2); $MAXS_{t}$ = the maximum allocated surface water amount in sub-region j in period t (m^3); $MAXG_{t}$ = the maximum allocated groundwater amount in sub-region j in period t (m^3); $RDP_{ijt}^{\pm}$ = the unit irrigation demand for crop i in sub-region j in period t (m^3/km^2); $MAXWA_{\pm t}$ = the maximum water amount allocated to agriculture in period t (m^3); $MAXWI_t^{\pm}$ = the maximum water amount allocated to industry in period t (m^3); $MAXWT_{maxt}^{\pm}$ = the maximum water amount allocated to tourism in period t (m^3); $MAXWH_{maxt}^{\pm}$ = the maximum water amount allocated to household in period t (m^3); $MAXUT_t^{\pm}$ = total wastewater treatment capacity in period t (tonne); $COSL_{ijt}^{\pm}$ = amount of soil loss from the land planted with crop i in sub-region j in period t (kg/km^2); $MAXCSL_{\pm t}$ = the allowed amount of soil loss in period t (kg); $QN_{ijt}^{\pm}$ = nitrogen percent content of the soil in sub-region j in period t (%); $QNI_{\pm kjt}$ = unit nitrogen discharge by industry k in sub-region j in period t ($tonne/m^3$); $QNT_{\pm jt}$ = unit nitrogen discharge by tourism industry in sub-region j in period t ($tonne/m^3$); $QNR_{\pm jt}$ = unit nitrogen discharge by household in sub-region j in period t ($tonne/m^3$); $NRE_{\pm t}$ = nitrogen removal efficiency in period t (%); $MAXTN_{\pm t}$ = the allowed amount of nitrogen discharge in period t (kg); $QS_{ijt}^{\pm}$ = phosphorus percent content of the soil in sub-region j in period t (%); $QPI_{\pm kjt}$ = unit phosphor discharge by industry k in sub-region j in period t ($tonne/m^3$); $QPT_{\pm jt}$ = unit phosphor discharge by tourism industry in sub-region j in period t ($tonne/m^3$); $QPR_{\pm jt}$ = unit phosphor discharge by household in sub-region j in period t ($tonne/m^3$); $PRE_{\pm t}$ = phosphor removal efficiency in period t (%); $MAXTP_{\pm t}$ = the allowed amount of phosphor discharge in period t (kg).

Chance-constrained programming:

As mentioned before in the 2.1 Development of Interval Multi-Random Factorial Programming, the constructed model can be generalized as Chance-constrained programming problem that is formulated as follows:

when $p_i = 0.01$,

$$\sum_{j=1}^{n}[a_{ij} - 2.33 \cdot \sigma_{aij(\omega)}]x_{ij} \le b_i + 2.33 \cdot \sigma_{bi(\omega)}, \forall i \tag{S15}$$

when $p_i = 0.1$,

$$\sum_{j=1}^{n}[a_{ij} - 1.28 \cdot \sigma_{aij(\omega)}]x_{ij} \le b_i + 1.28 \cdot \sigma_{bi(\omega)}, \forall i \tag{S16}$$

Then, these two new transformed constraints are applied at Equation (S12) because of the random characteristics of the allowed amount of soil loss. According to this solution algorithm, the IMRFP model for water resources and farmland management in the Songhua River Watershed can be solved, and the corresponding lower and upper bounds of solutions can be obtained. The generated interval solutions can provide decision makers with multiple decision alternatives according to practical situations and preferences.