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Modeling and Stability Analysis of the Effect of Awareness Programs on the Control of Atmospheric Pollutants Emitted from Various Pollutant Emitting Sources to Reduce Global Warming

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ABSTRACT. This study explores the dynamics of atmospheric pollutants released from various sources, including vehicular traffic, small and large-scale production, building industries, and various other human activities, to lessen the threat posed by global warming. These pollutants are also responsible for causing severe respiratory ailments and numerous fatalities among human populations. The presence of various pollutants emitting sources has led to a rise in the cumulative concentration of pollutants in the atmosphere that pose a threat both to the environment and human health. Therefore, it is very important to study the mitigation of such pollutants by making people aware of their harmful effects. Media awareness campaigns can have a significant impact on the mitigation of pollutants in the atmosphere causing global warming. Given this, in the present study, the role of media awareness campaigns in the abatement of pollutants is explored by developing and analyzing a nonlinear mathematical model. The stability theory of ordinary differential equations is used to analyze the mathematical model. To investigate the feasibility of the model system, local and global stability conditions are established. The model analysis demonstrates that media awareness programs have a significant impact on reducing air pollutants, which in turn reduces global warming.

Keywords: modeling, atmospheric pollutants, awareness programs, global warming, stability, numerical simulation

1. Introduction

Gaseous pollutants, such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), and fluorinated gases, also known as greenhouse gases (GHGs), are substances that contribute to the greenhouse effect and global warming by effectively trapping heat in the Earth's atmosphere. These pollutants are accountable for the escalation in the Earth's mean temperature, resulting in climate change and its corresponding consequences. Human activities such as fossil fuel combustion, deforestation, and industrial processes have substantially increased their concentrations. This elevated level of concentration of atmospheric pollutants contributes to an increase in global temperature and hence global warming. The increasing demands of the human population for food, water, etc., have necessitated an increase in the number of infrastructures to support these needs. This includes a wide range of systems, such as agricultural systems, water supply networks, transportation infrastructure, urban development, natural resource management, and energy-generating facilities. The establishment of new industries or increased capacity of existing industries, vehicles powered by fossil fuels, livestock farming, manure management, and various household activities have also increased the concentration of greenhouse gases in the atmosphere, contributing to climate change and global warming.

Apart from gaseous pollutants, particulate matter containing tiny solid and liquid droplets that remain suspended in the atmosphere also impact significantly climate change and global warming. The deposition of dark-coloured particles, such as black carbon (released as a result of the incomplete combustion of fossil fuels and biomass) onto the snow and ice surfaces has the potential to diminish their albedo, which refers to their ability to reflect sunlight. This phenomenon leads to increased absorption of sunlight and heat, accelerating the melting of glaciers and snow cover (Schmidt, 2011; Zhang et al., 2011; Yang et al., 2015; Takemura and Suzuki, 2019; Chen et al., 2021).

These pollutants have various implications for human health, the most significant of which is the escalation of current health issues and the emergence of new health hazards. The emission of various kinds of pollutants, responsible for the overall warming of planet Earth, has an impact on weather patterns, global temperatures, and ecosystems. These changes have both immediate and long-term implications on human health (Brunekreef and Holgate, 2002; Liang and Kosatsky, 2020; Naiyer and Abbas 2022; WHO, 2022a). In this regard, Duijndam et al. (2022) have observed that rising sea levels can lead to forced migra-

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tion and displacement, which can destabilize social systems and expose individuals to health risks in unfamiliar environments. Thomson and Stanberry (2022) have revealed that the distribution and behaviour of disease-carrying vectors, such as mosquitoes and ticks, can be affected by changes in temperature and humidity patterns. This may cause the spread of diseases like malaria, dengue fever, and Lyme disease to new regions. According to the Environmental Protection Agency, climate change raises the risk of sickness through rising temperatures, more frequent heavy rains, and the consequences of storms. Gastrointestinal disorders such as diarrhoea, effects on the neurological and respiratory systems of the body, and damage to the liver and kidneys are a few examples of health repercussions (EPA, 2023). The deteriorated air quality due to the presence of gaseous pollutants and particulate matter, released from the above-mentioned pollutant-emitting sources in the atmosphere, has also an adverse effect on human health (WHO, 2022b). Several ailments like cardiovascular risk, myocardial infarction, left ventricular hypertrophy, heart failure, respiratory diseases, and death in the human population are caused by gaseous pollutants and particulate matters (Genc et al., 2012; Leary et al., 2014; Bourdrel et al., 2017; Cohen et al., 2017; Zhang et al., 2019). Exposure to increased PM2.5 concentrations for both short and extended periods may raise the chance of developing acute nasopharyngitis, serious respiratory, and lung problems in humans (Zang and Batterman, 2013; Lelived et al., 2015; Kheirbek et al., 2016; Zhang et al., 2019; Tong et al., 2020).

Awareness among people through media awareness programs regarding air quality has the potential to reduce the adverse effects of gaseous pollutants and particulate matters. The state of being cognizant, educated, and knowledgeable about a specific issue or subject is referred to as awareness. This article pertains to comprehending the sources and consequences of atmospheric pollutants that contribute to the phenomenon of global warming through media awareness programs. Unawareness refers to the absence of cognizance or understanding concerning a particular subject matter. In this article, this may refer to communities or individuals who are ignorant of the sources and consequences of atmospheric pollutants contributing to global warming. Thus, media can be one of the most powerful tools in making the majority of the population aware of diseases and various risk perceptions. To make people aware of the impact of pollutants on humans and their environment, various awareness programs are executed by the government and other non-governmental organizations. These awareness programs provide necessary information for healthcare and also provide a platform that can be used by people, patients, and health professionals to spread knowledge about health-related issues (Grossberndt et al., 2021; Madhumati et al., 2021).

Many researchers have proposed mathematical models to examine how media awareness campaigns affect the spread of infectious illnesses (Cui et al., 2008a, b; Funk et al., 2009; Misra et al., 2011; Agarwal and Pathak, 2014; Collinson and Heffernan, 2014; Sharma and Misra, 2014; Misra and Verma, 2015; Misra et al., 2016; Agaba et al., 2017, Sundar et al., 2018; Rai et al., 2019). For example, Cui et al. (2008a) suggested a model to examine how media awareness campaigns affect the transmission of infectious diseases by taking into account this effect through the infective population. A model was proposed by Misra et al. (2011) to examine the impact of media-sponsored awareness campaigns, by considering media as a separate variable in the set of equations in the model. They have shown that media plays a significant role in controlling infectious diseases. In addition, Misra et al. (2016) proposed and analyzed a mathematical model to investigate the function of farmer awareness in the mitigation of algal bloom and found that farmer awareness is a key component for the mitigation of algal bloom in the lake. It has also been demonstrated that the rate of implementation of awareness programs through the media has a significant impact on the reduction of carbon dioxide, which causes global warming (Sundar et al., 2018).

Very few studies have been conducted to investigate the dynamics of atmospheric pollutants from various sources that cause global warming using nonlinear mathematical models (Misra and Verma, 2014, 2017, 2022; Shukla et al., 2015, 2020, 2022; Goyal and Shukla, 2018; Sundar et al., 2019, 2021). By conducting a comprehensive analysis of the system's stability, our objective is to reveal the mechanisms through which awareness campaigns can efficiently aid in the mitigation of atmospheric contaminants. This, in turn, will foster a sustainable approach to combating global warming.

In view of the above, in this paper, we propose a nonlinear mathematical model to study the effect of awareness programs on the mitigation of pollutants released from population density-dependent pollutant emitting sources that contribute to global warming. To develop the model, the following factors are taken into account: (i) The human population density is governed by a logistic model; (ii)There are two subclasses of humans: the unaware class and the aware class; (iii) The cumulative density of pollutants emitting sources such as vehicular traffic, household uses of fuels, small-scale industries, etc., in the habitat is dependent on the density of the human population; (iv) The cumulative concentration of pollutants is dependent on emissions both from natural and anthropogenic pollutant-emitting sources; (v) The growth of the variable representing the awareness program is proportional to the cumulative concentration of pollutants.

2. Mathematical Model

To model the phenomenon, we assume that the region under consideration is affected by the pollutants emitted from the human population density-dependent sources of emission. $N(t)$ represents the human population density, $S(t)$ represents the cumulative density of pollutants emitting sources, and $C(t)$ represents the cumulative concentration of pollutants. Media awareness program is governed by a separate variable $M(t)$. The aware population $N_a(t)$ and the unaware population $N_u(t)$ are two subpopulations that make up the total population.

An aware population is defined as the segment of the population that knows all about the evils of environmental pollution. The growth of the human population density is represented using a logistic model, where the intrinsic growth rate is r and the carrying capacity is K. It is assumed that the population decreases with the rate coefficient $r_0 > 0$ due to natural factors. It is further assumed that the unaware people become aware due to awareness programs by the media and join the class of the aware population, whereas some of the individuals of the aware class also re-enter the unaware class due to forgetfulness. Due to the adverse effects of pollutants on human health, the human population density is assumed to decrease with the death rate coefficient $d > 0$. In view of these assumptions, the dynamics of the unaware population are described by the following ordinary differential equation:

$$\frac{dN_u}{dt} = r(N - \frac{N^2}{K}) - r_0 N_u - dN_u C - \theta N_u M + \theta_0 N_a \tag{1}$$

The unaware human population can be made more environmentally conscious and informed about the consequences of their actions through awareness programs through various forms of media. The constant $\theta > 0$ is the rate by which unaware people become aware due to awareness programs and $\theta_0 > 0$ is the rate by which aware people lose their awareness due to forgetfulness and rejoin the unaware class. The dynamics of the aware class are given in the following equation:

$$\frac{dN_a}{dt} = \theta N_u M - (r_0 + \theta_0) N_a - dN_a C \tag{2}$$

Unaware human populations can contribute to the growth of various sources of pollution. When individuals are unaware of the environmental repercussions of their actions and lifestyles, they may indulge in polluting activities. These activities include the excessive use of fossil fuels, improper refuse disposal, deforestation, and other related actions, as discussed above. Insufficient knowledge and understanding can result in actions that worsen pollution and its adverse impacts on the environment and public health. Therefore, in the third equation, the cumulative density of various pollutants emitting sources is taken to be proportional to the density of the unaware human population with a growth rate coefficient $\lambda > 0$. The constant $\lambda_0 > 0$ is the coefficient of the natural depletion rate of the sources due to improper working or being outdated and hence excluded from the system, as given below:

$$\frac{dS}{dt} = \lambda N_u - \lambda_0 S \tag{3}$$

Pollutants associated with global warming are produced by both human activities (such as burning fossil fuels) and natural sources (such as wetlands and volcanoes). Human activities are the primary cause of recent increases in greenhouse gas emissions, which have led to climate change. Therefore, it is reasonable to assume that the cumulative concentration of pollutants can be roughly proportional to the density of pollutants emitting sources. This assumption simplifies the modeling and analysis of the proposed environmental issue. The cumulative concentration of pollutants is modeled in the fourth equation, in which Q_0 represents the constant emission rate of pollutants in the atmosphere through natural sources. The constant $\delta > 0$ is the growth rate coefficient of pollutants due to pollutant emitting sources. The constant $\delta_0 > 0$ is the depletion rate coefficient of pollutants due to some natural factors, as shown below:

$$\frac{dC}{dt} = Q_0 + \delta S - \delta_0 C \tag{4}$$

It is noted that increasing environmental awareness and education can result in more responsible and sustainable behaviour. When individuals are aware of the consequences of their actions, they are more likely to reduce pollution and mitigate its effects. Consequently, resolving unawareness through media awareness programs is essential to environmental conservation and pollution reduction initiatives. The implementation of the awareness campaigns through many media platforms can significantly contribute to enhancing environmental consciousness and knowledge among uninformed human populations on the repercussions of their behaviours. Various forms of media, such as television, radio, newspapers, social media, documentaries, and internet platforms, have significant impacts on people's daily lives. The implementation rate of awareness programs by the media is represented by the variable M in the fifth equation. The constant $\phi > 0$ is the implementation rate of awareness programs and $\phi_0 > 0$ is the depletion rate coefficient due to the ineffective working of some programs:

$$\frac{dM}{dt} = \phi C - \phi_0 M \tag{5}$$

where $N_u(0) > 0$, $N_a(0) \geq 0$, $S(0) \geq 0$, $C(0) \geq 0$, and $M(0) \geq 0$.

Since the total population $N = N_a + N_u$, the above Equations (1) ~ (5) reduces to:

$$\frac{dN}{dt} = r(N - \frac{N^2}{K}) - r_0 N - dNC \tag{6}$$

$$\frac{dN_a}{dt} = \theta\left(N - N_a\right) M - (r_0 + \theta_0) N_a - dN_a C \tag{7}$$

$$\frac{dS}{dt} = \lambda\left(N - N_a\right) - \lambda_0 S \tag{8}$$

$$\frac{dC}{dt} = Q_0 + \delta S - \delta_0 C \tag{9}$$

$$\frac{dM}{dt} = \phi C - \phi_0 M \tag{10}$$

It is noted that, in the absence of various pollutants emitting sources, the cumulative concentration of global warming pollutants is Q_0 / δ_0.

To analyze the Equations (6) ~ (10), we need the following region of attraction:

$$\Omega = \{(N, N_a, S, C, M) \in R^5 : 0 \le N_a \le N \le K,\ 0 \le S \le S_m,\ \frac{Q_0}{\delta_0} \le C \le C_m,\ 0 \le M \le M_m \tag{11}$$

where $S_m = \lambda K / \lambda_0$, $C_m = 1 / \delta_0 \times (Q_0 + \delta\lambda K / \lambda_0)$, $M_m = \phi C_m / \phi_0$. The set draws all solutions that originate within the interior of the positive orthant.

Remark 1: For the feasibility of the Equations (6) ~ (10), $r - r_0 - dC > 0$ (from Equation (6)) for all time $t > 0$, being the growth rate of the human population. This condition is essential for maintaining a meaningful and realistic interpretation of the model behaviour and its representation of population dynamics.

3. Equilibrium Analysis

The Equations (6) ~ (10) has the following two equilibria: (i) E_0 (0, 0, 0, Q_0 / δ_0, $\phi Q_0 / \phi_0 \delta_0$). The existence of equilibrium E_0 is obvious. This implies that in the absence of the human population and subsequent absence of pollutants emitting sources, the cumulative concentration of atmospheric pollutants in the atmosphere remains at its natural level. Additionally, the level of media awareness programs about these issues will also be stable. (ii) $E^*(N^*, N_a^*, S^*, C^*, M^*)$. This equilibrium point implies that the cumulative density of pollutants emitting sources is dependent on the human population. As a result, the cumulative concentration of atmospheric pollutants contributing to global warming is higher than their natural levels. The increase in atmospheric pollutants affects the density of the human population significantly. This implies that the presence of higher levels of pollutants has consequences for human populations, in terms of health and other environmental impacts. Some media awareness programs can effectively control the emission of pollutants. This implies that raising awareness through media can potentially lead to a reduction in activities that emit pollutants. The existence of the equilibrium (ii) is given below.

The equilibrium values of different variables in equilibrium (ii) are given by the following algebraic equations:

$$r(1 - \frac{N}{K}) - r_0 - dC = 0 \tag{12}$$

$$\theta(N - N_a)M - (r_0 + \theta_0)N_a - dN_a C = 0 \tag{13}$$

$$\lambda(N - N_a) - \lambda_0 S = 0 \tag{14}$$

$$Q_0 + \delta S - \delta_0 C = 0 \tag{15}$$

$$\phi C - \phi_0 M = 0 \tag{16}$$

From above equations, we get:

$$a_1 C^2 - b_1 C - c_1 = 0 \tag{17}$$

$$\begin{aligned} a_1 &= \frac{\delta\lambda K}{\lambda_0 r} d^2 + \delta_0 \left(\frac{\theta\phi}{\phi_0} + d \right) > 0, \\ b_1 &= Q_0 (\frac{\theta\phi}{\phi_0} + d) + \frac{\delta\lambda K}{\lambda_0 r} d(r - 2r_0 - \theta_0) + \delta_0 (r_0 + \theta_0), \\ c_1 &= (Q_0 + \frac{\delta\lambda K}{\lambda_0 r}(r - r_0))(r_0 + \theta_0) > 0 \end{aligned} \tag{18}$$

From Equation (17), we get:

$$C = \frac{b_1 + \sqrt{b_1^2 + 4a_1 c_1}}{2a_1} > 0 \tag{19}$$

Thus, Equation (17) has a unique positive root $C = C^*$ in $Q_0 / \delta_0 \le C \le 1/\delta_0\ (Q_0 + (\delta\lambda / \lambda_0)K)$. Putting the value of $C = C^*$ in Equations (12) and (14) ~ (16), we can find the other equilibrium components of $E^*(N^*, N_a^*, S^*, C^*, M^*)$.

Remark 2. Differentiating Equation (17) with respect to ϕ, we get $dC / d\phi < 0$ in $Q_0 / \delta_0 \le C \le 1 / \delta_0(Q_0 + (\delta\lambda / \lambda_0)K)$. This derivation suggests that as the rate of implementation of media awareness programs increases, the cumulative concentration of pollutants in the atmosphere decreases. As a result, this reduction in pollutant levels contributes to a decline in global warming. This means the more effective and pervasive these media-based awareness campaigns become, the more they contribute to reducing pollutant emissions from various sources, which results in a discernible reduction in the phenomenon of global warming.

4. Stability Analysis

In this section, a study of the stability of equilibria is carried out in order to have a better understanding of the practicability of the Equations (6) ~ (10). A dynamical system is regarded as locally asymptotically stable when trajectories that start in close proximity to an equilibrium point remain bounded and approach that equilibrium point gradually as time progresses. On the other hand, a system is characterized as globally asymptotically stable if trajectories originating from any point in its state space converge over time to a single equilibrium point. This level of stability ensures convergence from any conceivable initial condition. In order to analyze the local stability properties of the equilibria, we calculate the Jacobian matrix for the model system described by Equations (6) ~ (10):

$$\mathbf{J} = \begin{bmatrix} r(1 - \frac{2N}{K}) - r_0 - dC & 0 & 0 & -dN & 0 \\ \theta M & -\begin{pmatrix} \theta M + \\ r_0 + \theta_0 \\ + dC \end{pmatrix} & 0 & -dN_a & \theta(N - N_a) \\ \lambda & -\lambda & -\lambda_0 & 0 & 0 \\ 0 & 0 & \delta & -\delta_0 & 0 \\ 0 & 0 & 0 & \phi & -\phi_0 \end{bmatrix} \tag{20}$$

Clearly, the Jacobian matrix J_0 corresponding to equilibrium $E_0(0, 0, 0, Q_0 / \delta_0, \phi Q_0 / \phi_0\delta_0)$ has one eigenvalue $r - r_0 - d(Q_0 / \delta_0)$ positive and hence E_0 is unstable in N direction and stable manifold in $N_a - S - C - M$ space.

***Proposition* 1.** The equilibrium E^* is locally asymptotically stable under the following conditions (see Appendix A in the Supporting Information):

$$\delta^2 < \frac{1}{3}\lambda_0\delta_0 \tag{21}$$

$$\lambda^2 < \frac{1}{3}\min\left[\frac{4}{9}\frac{r^2\lambda_0\delta_0}{K^2d^2},\ \frac{1}{3}\lambda_0(\theta M^* + r_0 + \theta_0 + dC^*)^2\psi\right] \tag{22}$$

***Proposition* 2.** The equilibrium E^* is nonlinearly asymptotically stable if the following conditions (see Appendix B in the Supporting Information) hold Ω:

$$S_1 = \frac{1}{3}\lambda_0\delta_0 - \delta^2 > 0 \tag{23}$$

$$S_2 = \frac{1}{3}\min[\frac{4}{9}\frac{r^2\lambda_0\delta_0}{K^2d^2},\ \frac{1}{3}\lambda_0(r_0+\theta_0)^2\psi_1] - \lambda^2 > 0 \tag{24}$$

These propositions imply that when the parameters δ and λ take on extremely small values, the possibility of satisfying Equations (21) ~ (24) increases. From this, it is clear that these particular factors destabilize the model system. This insight provides a vital understanding of how the values of these parameters affect the overall model behaviour and stability. This means that when pollutant-emitting sources speed up the growth rate of global warming pollutants, it becomes less likely that the above-mentioned stability conditions will be met. This causes the cumulative concentration of global warming pollutants to rise, which can have significant adverse effects on the human population.

5. Numerical Simulation and Discussion

In this section, to illustrate the feasibility of our analytical findings regarding local and nonlinear stability behaviour of interior equilibrium, we have performed the numerical simulation for the Equations (6) ~ (10) using MAPLE by choosing the following set of parameters: $Q_0 = 10$, $r = 0.6$, $K = 10000$, $r_0 = 0.001$, $d = 0.00001$, $\lambda = 0.15$, $\lambda_0 = 0.2$, $\delta = 0.001$, $\delta_0 = 0.05$, $\phi = 0.01$, $\phi_0 = 0.25$, $\theta = 0.05$, $\theta_0 = 0.5$. The equilibrium values of $E^*(N^*, N_a^*, S^*, C^*, M^*)$ are $N^* = 9{,}938.04843$, $N_a^* = 5{,}157.42252$, $S^* = 3{,}585.46943$, $C^* = 271.70938$ and $M^* = 10.86837$.

The eigenvalues of the Jacobean matrix corresponding to $E^*(N^*, N_a^*, S^*, C^*, M^*)$ of Equations (6) ~ (10) are –1.04661, –0.59626, –0.06456, –0.15923, and –0.2767. Since all the eigenvalues are negative, therefore, the internal equilibrium $E^*(N^*, N_a^*, S^*, C^*, M^*)$ is locally asymptotically stable. To exhibit the nonlinear stability of E^* in N-N_a-S space, the solution trajectories are shown in Figure 1a with different initial starts. It may be deduced from the figure that all solution trajectories beginning inside the region of attraction approach the equilibrium value. Further, in Figure 1b, we have shown the variation of nonlinear stability conditions S_1 [as given in Equations (23)], with respect to crucial parameters. It is clear from Figure 1b that S_1 remains positive for $\delta < 0.0577$ and it is negative for $\delta > 0.0577$. This implies that the stability condition is satisfied for $0 < \delta < 0.0577$ and for higher values of δ it will not be satisfied indicating that δ has a destabilizing effect on the model system. The variation of the cumulative density of pollutant-emitting sources (S), the cumulative concentration of pollutants emitted from these sources (C), and the density of awareness programs through media (M) with time t for distinct values of λ, the growth rate coefficient of pollutants emitting sources due to human population, is illustrated in Figures 2a, 2b and 2c, respectively. It is noted from Figure 2a that the cumulative density of pollutants emitting sources increases as λ increases. Since the emission of pollutants is dependent on pollutant-emitting sources and therefore the cumulative concentration of pollutants increasees as λ increases (Figure 2b). As such the density of media awareness programs also increases as λ increases (Figure 2c).

The variation of the cumulative concentration of pollutants (C), the density of the human population (N) and the density of awareness programs through media (M) with time t is plotted for distinct values of δ, the growth rate coefficient of the cumulative concentration of pollutants due to the emitting sources, is shown in Figure 3. In Figure 3a, the variation of the cumulative concentration of pollutants C with time t is plotted for different values of δ, the growth rate coefficient of cumulative concentration of pollutants due to pollutants emitting sources. It is found that as the growth rate of pollutants emitting sources increases, the cumulative concentration of pollutants also increases in the atmosphere. Pollutants can detrimentally affect human population density by exacerbating health problems, diminishing overall welfare, increasing mortality, and potentially inducing population migration or displacement from polluted regions. Therefore, the variation of human population density N with time t for different values of δ is plotted in Figure 3b. It is found that as the growth rate of pollutants emitting sources increases, the cumulative concentration of pollutants in the atmosphere increases, due to which the density of the human population in a habitat decreases. Therefore, it is crucial to increase the density of awareness programs through media. This implies that raising awareness through media channels is seen as a key strategy to control and mitigate the increased concentration of pollutants (Figure 3c).

As stated in the introduction, an increase in the cumulative concentration of pollutants in a habitat's environment has detrimental effects on the human population and may even result in fatalities. This effect is shown in Figure S1, where the variation in density of the human population is plotted with time t for different values of its death rate coefficient due to pollutants. In Figure S2, the variation of the cumulative concentration of pollutants C with time t is plotted for different values of ϕ, the implementation rate of awareness programs. It is shown that as the implementation rate of awareness programs increases, the cumulative concentration of pollutants decreases in the atmosphere.

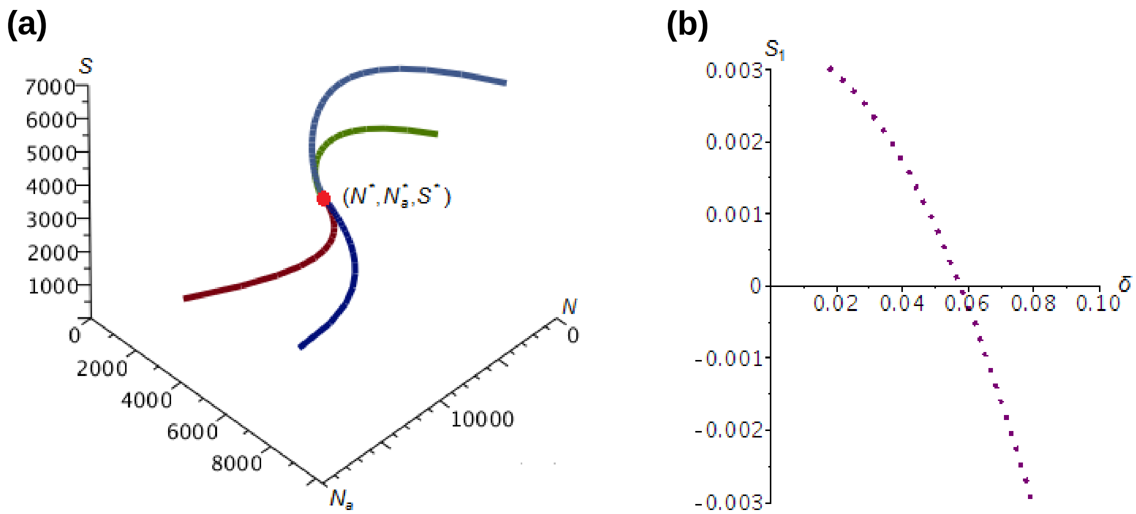

Figure 1. (a) Nonlinear stability in $N - N_a - S$ space; (b) Variation of stability condition S_1 with δ.

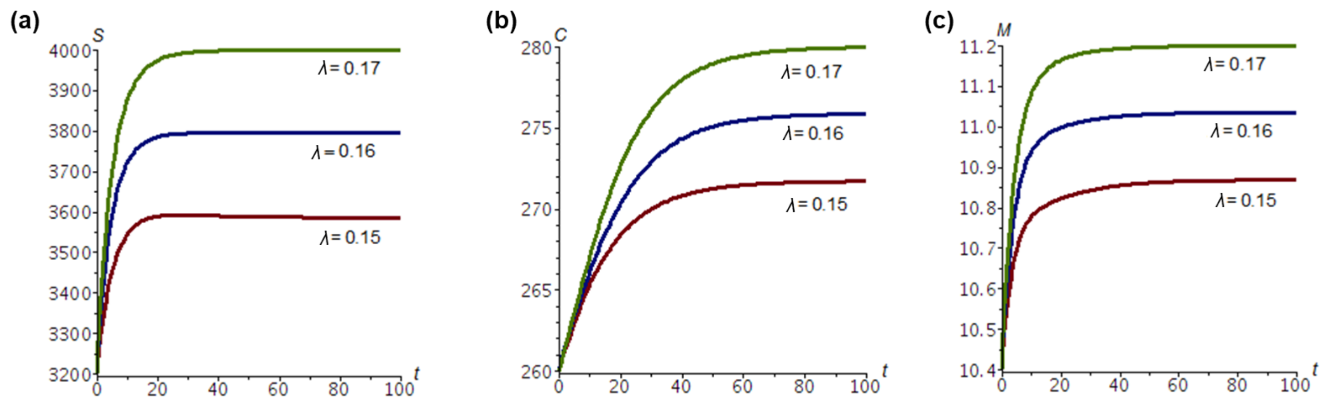

Figure 2. Variation of S (a), C (b), and M (c) with time t for different values of λ.

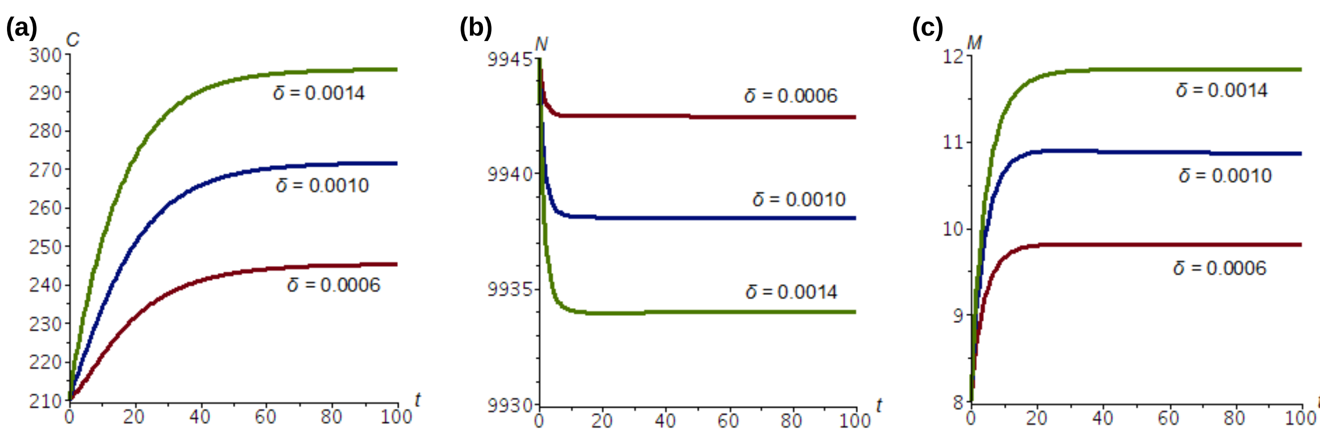

Figure 3. Variation of C (a), N (b), and M (c) with time t for different values of δ.

6. Conclusions

This paper presents a nonlinear mathematical model intended to investigate the effect of media awareness campaigns on reducing pollutants that contribute to global warming. The focus of the investigation is on pollutants emitted by various sources, including automobile traffic. The model includes five significant variables: human population density, aware population density, cumulative density of pollutant emitting sources, cumulative concentration of pollutant (including gaseous and particulate components), and a variable representing the effectiveness of media awareness programs. By establishing both local and global stability conditions, the study ensures a thorough investigation into the feasibility of the model system. The analysis has led to the following conclusions:

(i) The cumulative concentration of atmospheric pollutants increases as the rate of pollutants emitting sources increases. (ii) The density of the human population is adversely affected by the cumulative concentration of atmospheric pollutants. (iii) The model analysis confirms that media awareness programs effectively contribute to the reduction of atmospheric pollutants from various sources, emphasizing their usefulness in combating global warming. (vi) It is further demonstrated that as the implementation rate of media awareness programs increases, the cumulative concentration of pollutants decreases significantly, resulting in a reduction in global warming.

Thus, the present study highlights the importance of media awareness campaigns in reducing the concentration of pollutants that contribute to global warming. Using a robust mathematical model and stability analysis, the study demonstrates the efficiency of media awareness programs to combat the population growth-related increase in pollutant emissions.

In view of the above, the model analysis suggests that the implementation of media awareness programs is an effective control mechanism for atmospheric pollutants, which can lead to a decrease in global warming. In this regard, the model is very advantageous.

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