Supporting Information

Modeling and Stability Analysis of the Effect of Awareness Programs on the Control of Atmospheric Pollutants Emitted from Various Pollutant Emitting Sources to Reduce Global Warming

S. Sundar[1, *], N. Swaroop[2], R. Naresh[3]

[1] Department of Mathematics, Pranveer Singh Institute of Technology, Kanpur 209305, India
[2] Department of Mathematics, Christ Church College, Kanpur 208001, India
[3] Department of Mathematics, School of Basic & Applied Sciences, Harcourt Butler Technical University, Kanpur 208002, India

Appendix A

Proof of the proposition 1

To establish the local stability of E^*, let us consider the following positive definite function,

$$V = \frac{1}{2} l_1 N_1^2 + \frac{1}{2} l_2 N_{a1}^2 + \frac{1}{2} l_3 S_1^2 + \frac{1}{2} l_4 C_1^2 + \frac{1}{2} l_5 M^2 \quad \text{(SA1)}$$

where N_1, N_{a1}, S_1, C_1, M_1 are the small perturbations about $E^*(N^*, N_a^*, S^*, C^*, M^*)$ as described below:

$$N = N^* + N_1$$

$$N_a = N_a^* + N_{a1}$$

$$S = S^* + S_1$$

$$C = C^* + C_1$$

$$M = M^* + M_1$$

l_i (i=1, 2, 3, 4, 5) are positive constants to be chosen appropriately.

After differentiation of (SA1), using the linearized system of the model (6) ~ (10) and on simplification, we have:

$$\frac{dV}{dt} = -l_1 \frac{rN^*}{K} N_1^2 - l_2(\theta M^* + r_0 + \theta_0 + dC^*) N_{e1}^2 - l_3 \lambda_0 S_1^2 - l_4 \delta_0 C_1^2 - l_5 \phi_0 M_1^2 + l_2 \theta M^* N_1 N_{al} + l_3 \lambda N_1 S_1 - l_1 d N^* C_1 N_1 - l_3 \lambda N_{al} S_1 - l_2 d N_a^* C_1 N_{a1} + l_2 \theta (N^* - N_a^*) M_1 N_{a1} + l_4 \delta S_1 C_1 + l_5 \phi C_1 M_1 \quad \text{(A2)}$$

After performing some algebraic calculations and choosing：

$$l_1 < \frac{\delta_0 r}{3Kd^2 N^*}$$

$$l_2 < (\theta M^* + r_0 + \theta_0 + dC^*)\psi$$

$$l_3 = 1 = l_4$$

$$l_s < \frac{\phi_0 \delta_0}{2\phi^2}$$

$$\psi = \min\left[\frac{1}{9}\frac{r^2\delta_0}{K^2 d^2 \theta^2 M^*}, \frac{1}{4}\frac{\delta_0}{d^2 N_a^{*2}}, \frac{1}{4}\frac{\delta_0 \phi_0^2}{\theta^2 \phi^2 \left(N^* - N_a^*\right)^2}\right]$$

$\frac{dv}{dt}$ will be negative definite under the Equations (19) and (20) showing that V is Lyapunov function. This proves the proposition.

Appendix B

Proof of the proposition 2

To establish the nonlinear stability of E^*, let us consider the following positive definite function:

$$W = p_1(N - N^* - N^* \log\frac{N}{N^*}) + \frac{1}{2}p_2(N_a - N_a^*)^2 + \frac{1}{2}p_3(S - S^*)^2 + \frac{1}{2}p_4(C_1 - C_1^*)^2 + \frac{1}{2}p_5(M - M^*)^2 \quad \text{(SB1)}$$

and p_i (i=1, 2, 3, 4, 5) are positive constants to be chosen appropriately.

Differentiating above equation with respect to t and using the Equations (6) ~ (10), we get:

$$\frac{dW}{dt} = -p_1\frac{r}{K}(N - N^*)^2 - p_2(\theta M + r_0 + \theta_0 + dC)(N_a - N_a^*)^2 - p_3\lambda_0(S - S^*)^2 - p_4\delta_0(C - C^*)^2 - p_5\phi_0(M - M^*)^2 +$$
$$p_2\theta M(N - N^*)(N_\alpha - N_\alpha^*) + p_3\lambda(N - N^*)(S - S^*) - p_1 d(N - N^*)(C - C^*) - p_3\lambda(N_a - N_a^*)(S - S^*) -$$
$$p_7 dN_a^*(N_a - N_a^*)(C - C^*) - p_2\theta(N^* - N_a^*)(N_a - N_a^*)(M - M^*) + p_4\delta(S - S^*)(C - C^*) + p_5\phi(M - M^*)(C - C^*) \quad \text{(SB2)}$$

After performing some algebraic manipulations and choosing,

$$p_1 = \frac{\delta_0 r}{3Kd^2}$$

$$p_2 < (r_0 + \theta_0)^2\psi_1$$
$$p_3 = p_4 = 1$$

$$p_5 < \frac{\phi_0\delta_0}{2\phi^2}$$
$$\psi_1 = Min[\frac{1}{9}\frac{r^2\delta_0}{K^2d^2\theta^2M^2}, \frac{1}{4}\frac{\delta_0}{d^2N_a^{*2}}, \frac{1}{4}\frac{\delta_0\phi_0^2}{\theta^2\phi^2(N^* - N_a^*)^2}]$$

$\frac{dw}{dt}$ will be negative definite inside the region of attraction Ω provided the conditions (21) and (22) are satisfied showing that W is Lyapunov function. This proves the proposition.

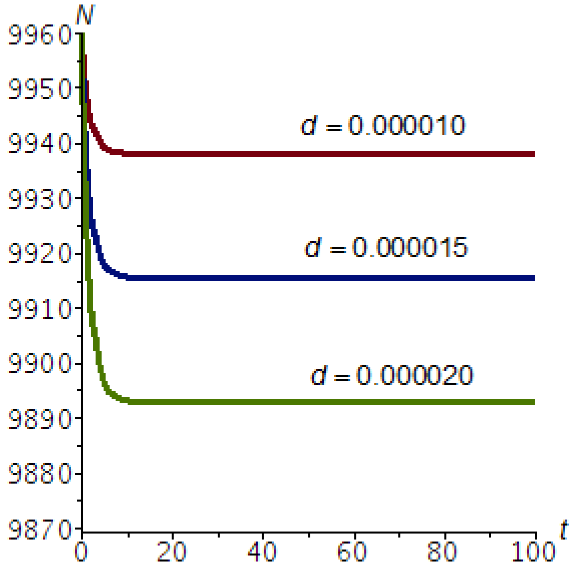

Figure S1. Variation of N with time t for different values of d

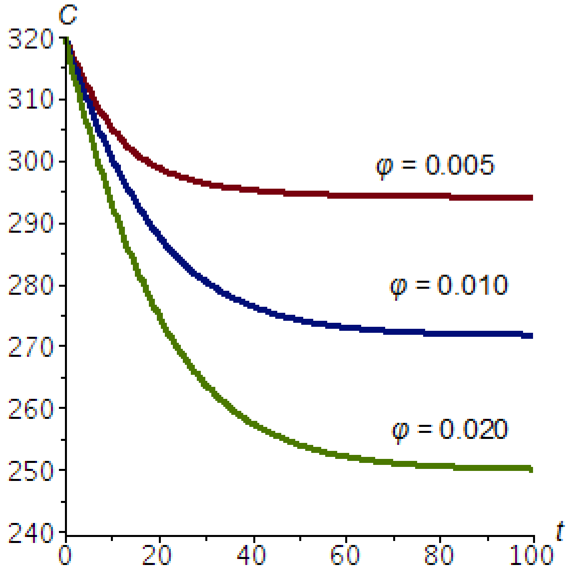

Figure S2. Variation of *C* with time *t* for different values of ϕ.